

Gauß-Elimination

$$\begin{array}{l}
 1x_1 + 2x_2 + 3x_3 = 10 \\
 \text{Zeile} - \frac{4}{1} \cdot 1. \text{ Zeile: } 4x_1 + 5x_2 + 6x_3 = 11 \\
 \text{Zeile} - \frac{7}{1} \cdot 1. \text{ Zeile: } 7x_1 + 8x_2 + 10x_3 = 70
 \end{array}$$

$$L^{(1)} = \begin{pmatrix} 1 & & \\ -\frac{4}{1} & 1 & \\ -\frac{7}{1} & & 1 \end{pmatrix}$$

$$\begin{array}{l}
 1x_1 + 2x_2 + 3x_3 = 10 \\
 -3x_2 - 6x_3 = -29 \\
 \text{Zeile} - \frac{-6}{-3} \cdot 2. \text{ Zeile: } -6x_2 - 11x_3 = 0
 \end{array}$$

$$L^{(2)} = \begin{pmatrix} 1 & & \\ & 1 & \\ & -\frac{-6}{-3} & 1 \end{pmatrix}$$

$$\begin{array}{l}
 1x_1 + 2x_2 + 3x_3 = 10 \\
 -3x_2 - 6x_3 = -29 \\
 +1x_3 = 58
 \end{array}$$

Rückwärtseinsetzen
liefert: $x_3 = 58$, etc

Beobachtung: $LU = \begin{pmatrix} 1 & & \\ +\frac{4}{1} & 1 & \\ +\frac{7}{1} & +\frac{-6}{-3} & 1 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 \\ -3 & -6 & \\ & & 1 \end{pmatrix} = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 10 \end{pmatrix}$