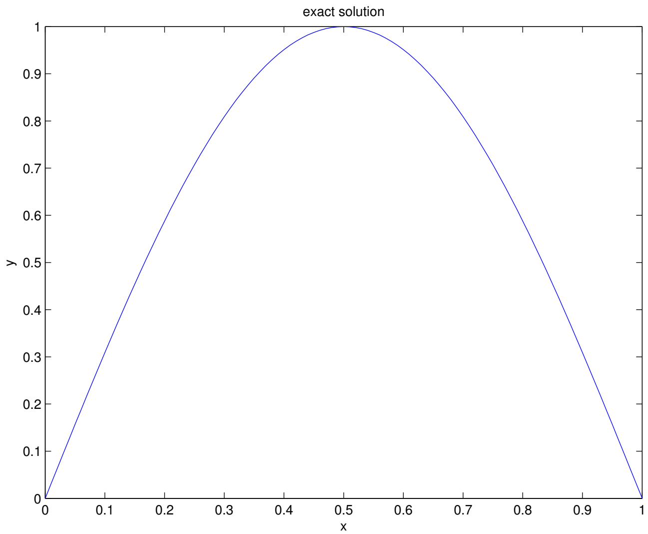
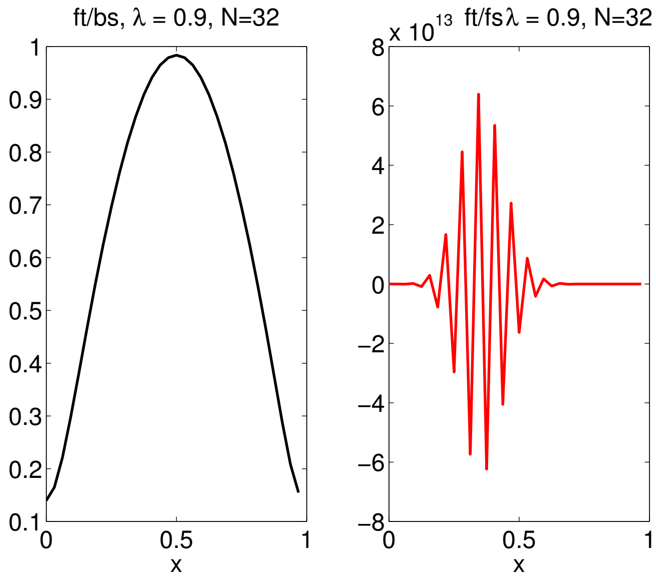
Advektionsgleichung

$$u_t + u_x = 0, \qquad x \in (0,1),\, t > 0, \qquad u(0,t) = u(1,t) \quad \forall t > 0$$

$$u(x,0) = \sin(\pi x), \qquad \text{periodische Randbedingungen: } u(0,t) = u(1,t) \text{ für alle } t > 0$$



exakte Lösung: $u(x,t) = \sin(\pi(x-t))$



Vergleich forward time/backward space und forward time/forward space

Advektionsgleichung, II

forward time/backward space: $$\frac{1}{k}\left(u_i^{n+1} - u_i^n\right) = \frac{a}{h}\left(u_i^n - u_{i-1}^n\right)$$

forward time/forward space: $$\frac{1}{k}\left(u_i^{n+1} - u_i^n\right) = \frac{a}{h}\left(u_{i+1}^n - u_i^n\right)$$

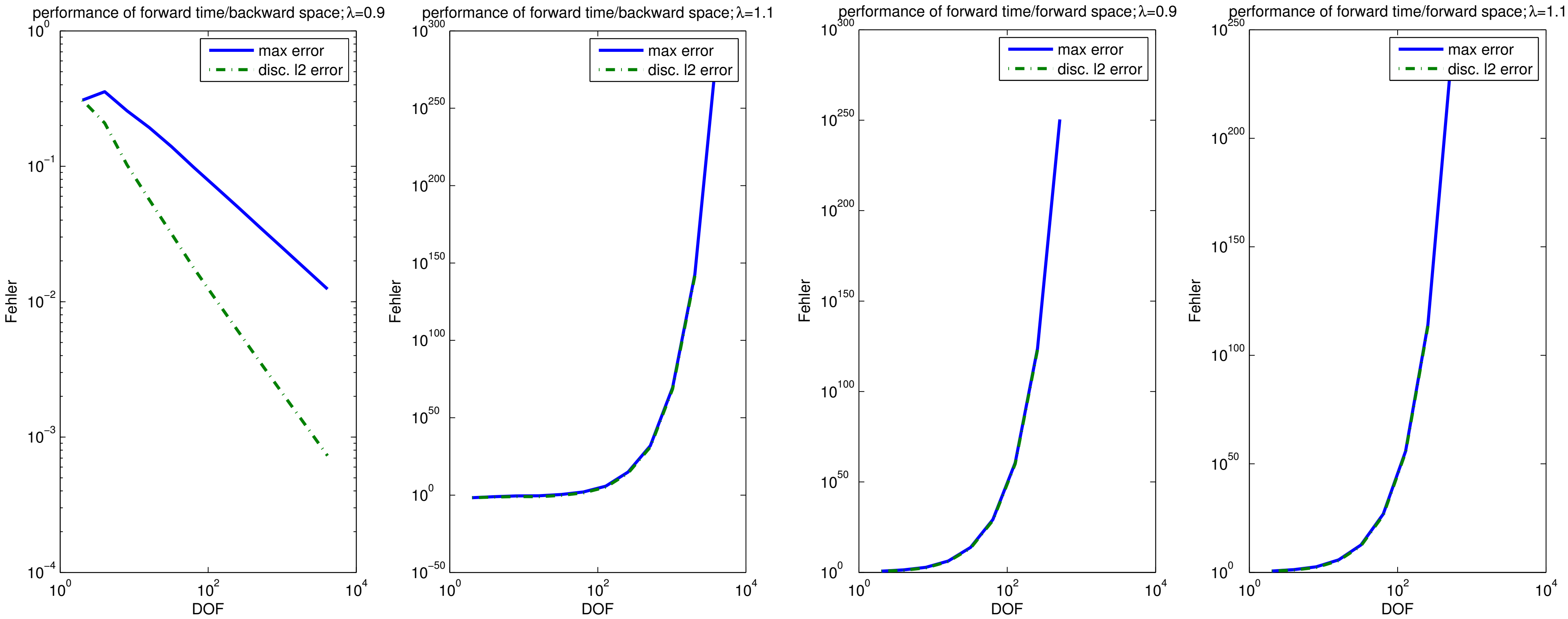
forward time/central space: $$\frac{1}{k}\left(u_i^{n+1} - u_i^n\right) = \frac{a}{2h}\left(u_{i+1}^n - u_{i-1}^n\right)$$

Lax-Friedrichs: $$\frac{1}{k}\left[u_i^{n+1} - \frac{1}{2}\left(u_{i+1}^n + u_{i-1}^n\right)\right] = \frac{a}{2h}\left(u_{i+1}^n - u_{i-1}^n\right)$$

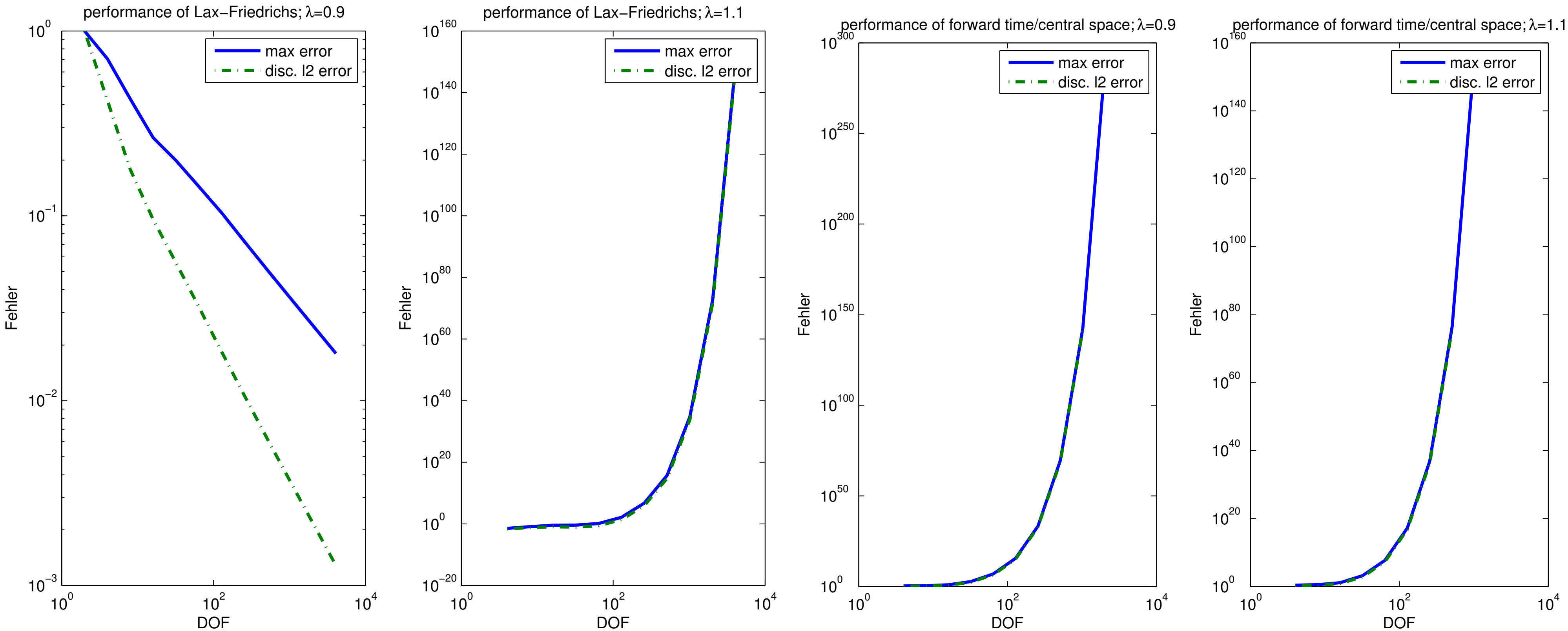
Leap Frog: $$\frac{1}{2k}\left[u_i^{n+1} - u_i^{n-1}\right] = \frac{a}{2h}\left(u_{i+1}^n - u_{i-1}^n\right)$$

- CFL-Bedingung $k|a|/h \leq 1$ ist notwendig bei (zeit-)expliziten Verfahren
- “upwinding” ist eine gut Idee

Advektionsgleichung: forward time/backward space & forward time/forward space



Advektionsgleichung: Lax-Friedrichs & forward time/central space



Advektionsgleichung: Leap Frog