

Vektoriteration

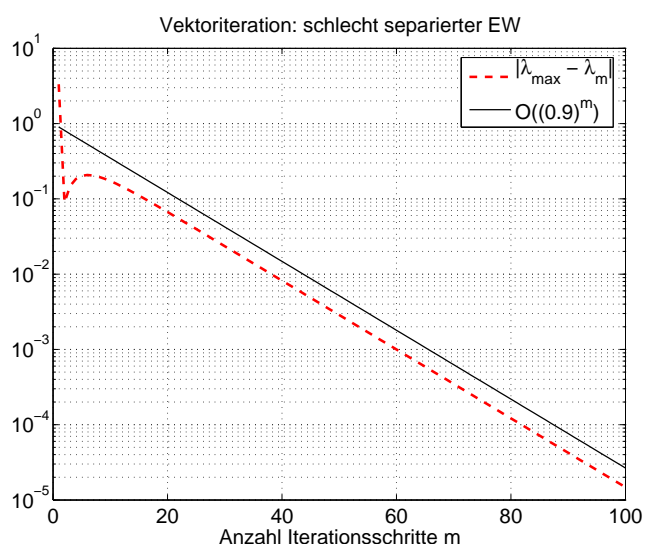
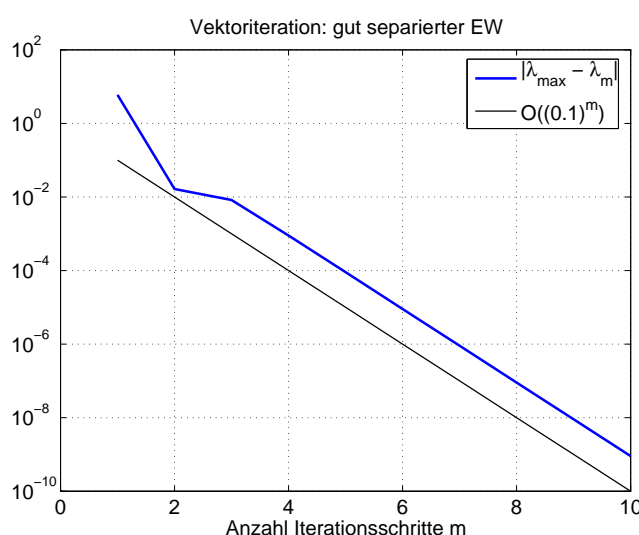
Iterationsvorschrift:

$$x_{l+1} := \frac{Ax_l}{\|x_l\|_2}$$

$$\tilde{\lambda}_{l+1} = x_{l+1}^\top Ax_{l+1}$$

$$A_1 = \begin{pmatrix} 10 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \lambda_1 = 10, \quad \lambda_2 = 1, \quad \lambda_3 = 0, \quad \left| \frac{\lambda_2}{\lambda_1} \right| = 0.1$$

$$A_2 = \begin{pmatrix} 10 & 1 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \lambda_1 = 10, \quad \lambda_2 = 9, \quad \lambda_3 = 0, \quad \left| \frac{\lambda_2}{\lambda_1} \right| = 0.9$$



$$A_3 = \begin{pmatrix} c & s & 0 \\ -s & c & 0 \\ 0 & 0 & 0.1 \end{pmatrix},$$

$$c = \cos(\pi/3), \quad s = \sin(\pi/3),$$

$$\lambda = 0.5 \pm 0.5\sqrt{3}, \quad \lambda = 0.1$$

Iterationszahl l	$\tilde{\lambda}_l$
1	0.366666666666667
2	0.49800995024876
3	0.49998000099995
4	0.49999980000010
5	0.49999999800000
6	0.49999999998000
7	0.49999999999980
8	0.50000000000000
9	0.50000000000000
10	0.50000000000000
11	0.50000000000000

In allen Beispielen ist $x_0 = (1, 1, 1)^\top$.

Inverse Iteration und Rayleighquotienteniteration

Inverse Iteration mit Shift λ :

$$\begin{aligned}\tilde{\lambda}_l &= x_l^\top A x_l \\ \tilde{x}_{l+1} &:= (A - \lambda)^{-1} x_l \\ x_{l+1} &:= \frac{\tilde{x}_{l+1}}{\|\tilde{x}_{l+1}\|_2}\end{aligned}$$

Rayleighquotientenmethode

$$\begin{aligned}\tilde{\lambda}_l &= x_l^\top A x_l \\ \tilde{x}_{l+1} &:= (A - \tilde{\lambda}_l)^{-1} x_l \\ x_{l+1} &:= \frac{\tilde{x}_{l+1}}{\|\tilde{x}_{l+1}\|_2}\end{aligned}$$

$$A = \begin{pmatrix} 10 & 1 & 0 \\ 1 & 9 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \lambda_1 \approx 10.6180, \quad \lambda_2 \approx 8.3820, \quad \lambda_3 = 0,$$

$x_0 = (1, -1, 0)^\top \rightsquigarrow$ Rayleighquotienteniteration konvergiert gegen λ_2

inverse iteration und Rayleighquotientenmethode

